SAMPLING UNCERTAINTY IN COORDINATE MEASUREMENT DATA ANALYSIS

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ABSTRACT

Coordinate metrology generates important software issues. After measurement data are collected in the form of position vectors, the data analysis software must derive the necessary information from the set of points. Uncertainty is an important issue in this data analysis. When extreme fit approaches are employed for form error evaluation, the uncertainty is closely related to the sampling of measurement points. Those measurement points are a subset of the true surface and, consequently, the extreme fit result differs from the true value. In this paper, we investigate the functional relationship between such an extreme fit uncertainty and the sampling of measurement points. The important questions to be answered are: what parameters affect the functional relationship, and how can this scheme be applied to sampling of measurement points. An analytic and experimental approach are presented for a flatness example.

INTRODUCTION

Three-dimensional metrology has brought a significant change in dimensional measurement. Compared to the traditional two point measurements, three-dimensional measurement yields more comprehensive information about the real part geometry. Three-dimensional metrology generates surface coordinates of a measured feature instead of measuring the geometric dimensions. Thus, the measurement output is a collection of digitized surface points. With these surface points, more details about surface variation can be observed, and various sections of different geometric features can be measured in a single process. Currently, coordinate measuring machines (CMMs) are widely employed as dimensional measurement devices.

After measurement data are collected by a three-dimensional measuring machine, an independent numerical analysis must be performed. This is an important characteristic and advantage of the three-dimensional metrology. Since the measurement data are in geometric form as a set of points, in order to obtain geometric parameters of part variation or to make a tolerance conformance decision, the raw data must be interpreted into a parametric form by an appropriate analysis procedure. Thus, three-dimensional metrology separates the measurement data analysis procedure as a software procedure from the measurement data collecting procedure, which is a hardware procedure. The data analysis has become one of the key components in three-dimensional metrology and is the focus of this paper.

An important issue associated with the data analysis procedure is that it is completely independent from the hardware measurement procedure and utilizes only measured coordinate points. For evaluation of a form error such as flatness as defined in ASME standard Y14.5 1,2, a minimum zone must be established from the measurement data. The minimum zone evaluation algorithm has been reported in various studies Etesami and Qiao3, Wang4, Huang, etc. al.5,6, Kanada and Suzuki7,8, Carr and Ferreira9,10. The problem is formulated as an extreme fit evaluation that searches for the extreme points that characterize the zone. However, such an extreme fit has limitations in accuracy. Unlike the least squares fit, that determines the fitting geometry using a maximum likelihood probability from all the measured points, the extreme fit is determined by a few extreme points, the same number of points as the geometry's degree of freedom. Thus, the result is highly sensitive to the extremes of the measured points. Since the measured points including the extreme points are a sample of the true surface, the measured points are not a complete copy of the true surface. Therefore, the evaluated results are not exactly equal to the ideal extreme fit, and the results vary as different samples are taken from the true surface. Thus, the extreme fit evaluation results have a level of uncertainty associated with sampling (Choi11).

Clearly, the question that must be answered is how the uncertainty of the evaluation is related to the sampling. The critical parameters for the uncertainty must be identified. Since the sample size is directly related to the measurement time and the data processing cost, the selection of the sample size is a primary issue in the coordinate metrology. It is obvious that an evaluation with more dense measurement data yields more accurate results, but a quantitative assessment must be made. Thus, it is necessary to find a sample size that will yield sufficiently accurate results.

The objective of this research is to investigate the functional relationship of the sampling parameters to the uncertainty in the extreme fit evaluation. We investigate how the properties of measurement data such as density, distribution, and the geometry cause uncertainty in the evaluation result. Such knowledge provides a fundamental basis for sampling strategy in the inspection planning stage. In this paper, the uncertainty in extreme fit evaluation is discussed and a generalized form error distribution model is presented. For the functional relationship, an analytic approximation model and a numerical model are proposed. The work is demonstrated with a flatness example.

RELATED WORK

Recognition of the sampling problem can be found in Hocken, et. al.12, 13, where extensive numerical experiments on data fitting algorithms were conducted to investigate the sampling issues. They tested least squares fit, min-max fit and minimum zone evaluations on various geometric primitives such as line, plane, circle, sphere, and cylinder. They generated simulated measurement data embedded with a characteristic form error such as tri-lobbing error for a circular feature. Then, they ran multiple sampling and fitting procedures and determined the variation of the evaluated parameters. The results were reported for the cases of a line, plane, circle, sphere and cylinder. With their results, they concluded that variation is substantially large with current practice sampling size.

Weckenmann et. al.14,15, also addressed sampling issues for coordinate metrology. They recognized that the extreme fit is more suitable for functionality evaluation but it yields variations that are sensitive to sampling. They investigated the effect of sample point location and sample size on a circular feature with tri-lobbing error and a linear feature with undulation form error. From their numerical experiments, they showed the uncertainty of the form error evaluation, represented as the dispersion of the evaluated value, decreases with increasing sample size.

Mestre and Abou-Kandil16 approached the problem from a different direction. Recognizing the problem with current extreme fit evaluations, which ignore the variation in unmeasured sites, they proposed a new method for the flatness evaluation. Employing Bayesian prediction theory, they derived the confidence interval of the surface variation in unmeasured sites. Then, the minimum zone was determined with respect to the derived confidence interval. By doing so, they could overcome the plug-in estimation error of extreme fit evaluation, and the ensuing estimated minimum zone was closer to the true flatness value.

As shown above, the sampling uncertainty problem in coordinate metrology has been recognized in previous studies, but a formal evaluation method has not been reported, and the results are limited to a specific cases. In this paper, we present a more general formulation of the sampling effect for the extreme fit. We consider the functional relationship between the sampling uncertainty and the form error type, the sample size, and the geometry factors. We also present a systematic methodology to investigate the functional relationship between the factors.

EXTREME FIT AND UNCERTAINTY

A typical form of the extreme problem is represented as

$$\min M(\theta) = \max [\text{distance}\{\mathbf{p}_i, S(\theta)\}] \quad (1)$$

where S is the fitted surface model, θ represents the model parameters, and $\mathbf{p}_i$ are measured points. Since the function M(θ) is non-differentiable, it must be determined by an iterative search algorithm. If the fitting geometry is properly defined, there is a unique solution that will satisfy the optimality condition and will be constrained by the extreme points. Thus, the fitting geometry is defined by a few extreme points of the measured point set.

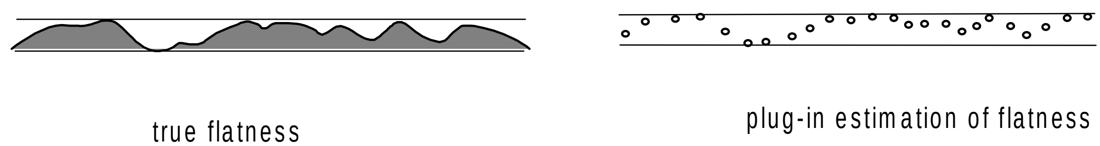

Figure 1 True Value vs. Estimated Value by Measured Data.

The uncertainty associated with extreme fit comes from the plug-in nature of the extreme fit. The plug-in estimation is the evaluation of a parameter with a finite number of samples regarding them as the entire population. The extreme fit geometry is determined from a few extreme points. However, the extreme points in the measurement data that contribute to the extreme fit geometry are not necessarily the extreme points in the true surface profile. As shown in Figure 1, the true flatness is defined as the minimum zone over the true surface profile. However, the flatness is evaluated from only the measured points and the extreme points in the measurement data are samples of the true surface points. Thus, the true flatness value and the flatness evaluated by measured points are not equivalent, because the measurement data fail to capture the true extreme points.

Depending on the data sampling strategy, the evaluated extreme fit values will vary. Thus, even though the extreme fit evaluation result is determined from the extreme points of the measured data, the result is not a perfectly true value. There is always a discrepancy between the true value and the extreme fit evaluation value because the measured data is a finite sample from an unknown true surface. Also the evaluated value varies by different samples. Figure 2 represents the flatness value evaluated from a surface with different samples. The samples are taken from a simulated surface that has uniform distribution of noise. For each sample group, the flatness is evaluated by a minimum zone evaluation method and the flatness value is plotted with respect to the sample group number. Even though the samples are taken from the same surface, the evaluated flatness value varies with different samples. Such a variation is due to the plug-in evaluation of the minimum zone evaluation, and it is the sampling uncertainty. Thus, when an extreme fit value is evaluated, the standard error of the evaluated value must be examined in order to consider the sampling uncertainty. More detail about evaluating the standard error from a measurement data set can be found in Choi11.

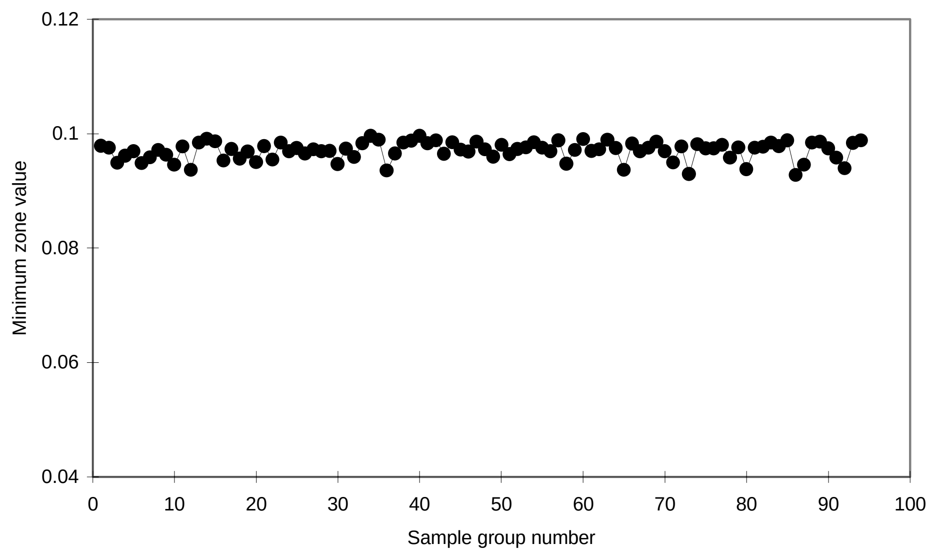

Figure 2 Variation of a Minimum Zone Value With Respect to Different Samples.

The focus of the paper is how the uncertainty is related to the property of the measured data. Clearly, the number of measured points is related to the uncertainty. As more data points are used for evaluating the fitting surface, the extreme fit surface is closer to the true value, since larger number of measurement points yields a more comprehensive sampling of the true surface. Figure 3 shows the relation between the sample size and the uncertainty. The same flatness example is tested with different numbers of samples. In each case, the standard error of the flatness value is derived. The standard error representing the uncertainty decreases with respect to the sample size.

Likewise, we need to investigate what other properties of measured data contribute to the uncertainty, and how are they linked together.

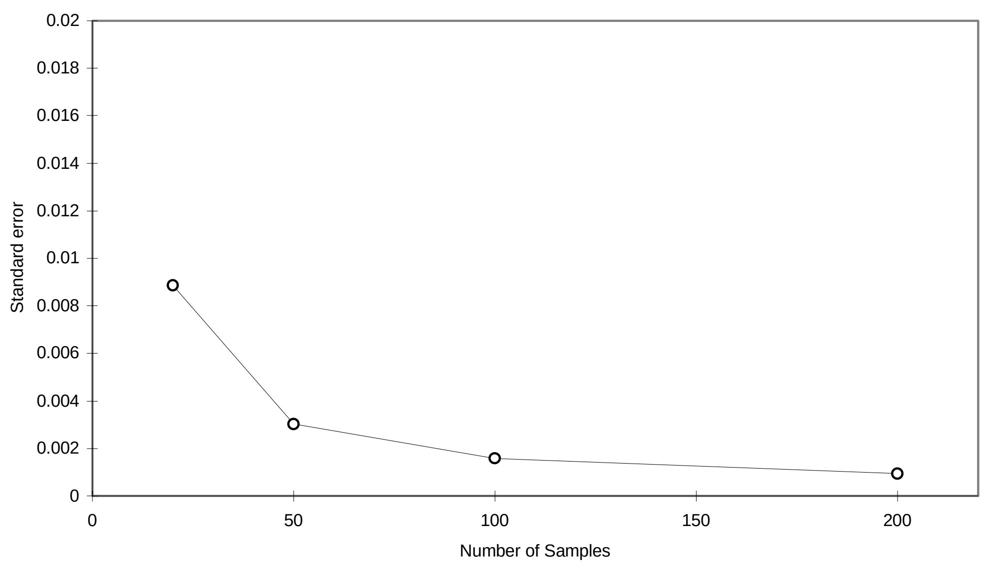

Figure 3. The Standard Error vs. Number of Sample.

DISTRIBUTION OF FORM ERROR

Form error is surface variation due to imperfect manufacturing. The form error of the surface is copied into the measured points, and it can be represented as the distribution of the probability density with respect to a fixed coordinate reference frame (Figure 4). In order to investigate the surface form error effect on the uncertainty, we need to model the form error distribution. A normal distribution is a typical assumption in a statistical situation. The majority of random distributions with large number of data, that are encountered in the real world, are close to the normal distribution. Even for engineering surfaces, form errors are often assumed to be normally distributed, but the actual distribution may seriously depart from normal distribution (Figure 5). The normal distribution assumption is appropriate when form error is affected by various small scale, random and independent effects. It is also assumed that mechanical properties must be uniform and stable. Actual engineering surfaces undergo various non-uniform errors caused by tool wear, variation of stiffness, heat treatment deformation, etc.. It is difficult to model such form errors with a normal distribution because the normal distribution is symmetric and has only two degrees of freedom.

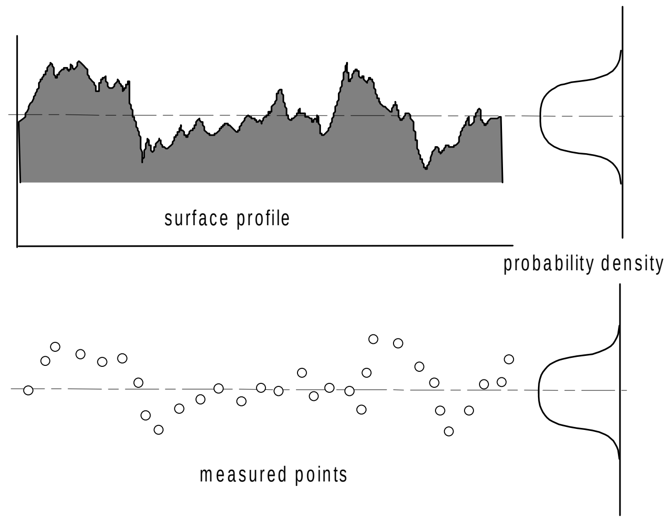

Figure 4. Form Error Distribution.

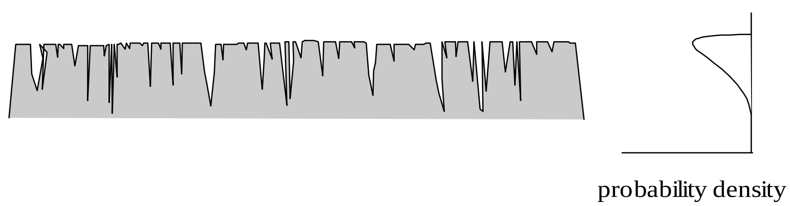

Figure 5 Typical Non-normal Engineering Surface.

In this paper, a more appropriate model for the form error distribution, beta distribution (Patel, J. et. al17), is employed. The beta distribution has a finite range as opposed to the infinite tails of the normal distribution that may not properly represent the form error whose variation is finite due to the material property. The beta distribution has four degrees of freedom and can be used to represent a wide range of distributions from uniform distribution to bell shaped distributions similar to normal distribution, as well as model asymmetric distributions. Thus, the beta distribution is flexible enough to take into account the actual variations in real surfaces. It has been shown that the beta distribution is more appropriate than the normal distribution for real surface form errors (Shunian, et. al.18).

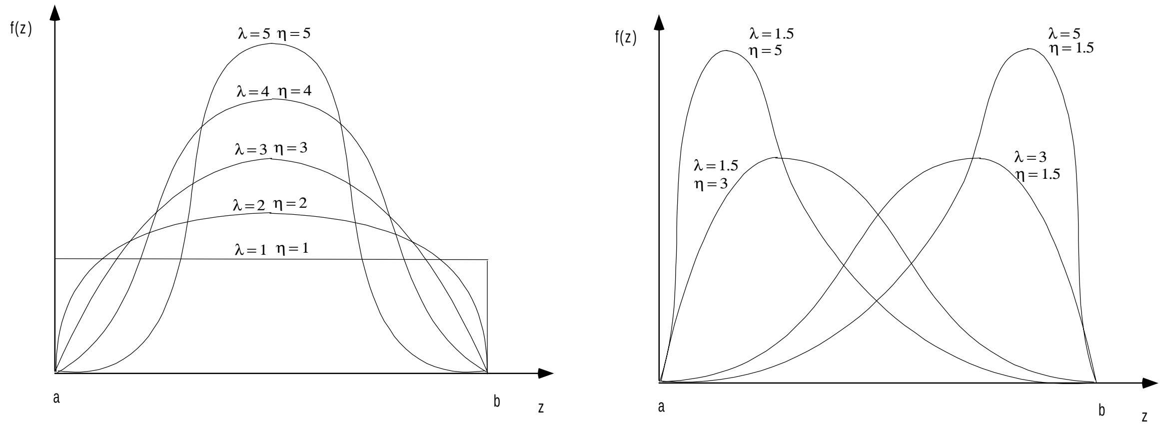

Figure 6 Beta Distribution.

The beta distribution can be formulated as follows. It is defined in a finite range [a,b]. The probability density function of a generalized beta distribution is defined with the exponents h and l as

$$f(x,\lambda,\eta,a,b)=\frac{1}{(b-a)B(\lambda,\eta)}\left(\frac{x-a}{b-a}\right)^{\lambda-1}\left(1-\frac{x-a}{b-a}\right)^{\eta-1} \tag{2}$$

$$0<\lambda,\ 0<\eta,\ a\le x\le b$$

where $B(\lambda,\eta)$ is the beta function defined as

$$B(\lambda,\eta)=\int_0^1 z^{-1}(1-z)^{\eta-1}dz \tag{3}$$

By substituting a = 0 and b = 1 in (3), the unit beta distribution can be defined as

$$f(z,\lambda,\eta)=\frac{1}{B(\lambda,\eta)}z^{\lambda-1}(1-z)^{\eta-1} \tag{4}$$

$$0<\lambda,\ 0<\eta,\ 0\le z\le 1$$

The function f represents the probability density of the surface area with respect to the height z. The height z represents the normalized surface form error level. The surface form error characteristic is defined by the beta-distribution exponents λ and η. When λ and η values are unity, the distribution is equal to a uniform distribution, and as the λ and

η values increase from unity, the distribution becomes similar to the normal distribution (Figure 6). λ and η are the weight of the distribution in each direction respectively. As the value of λ becomes larger, the distribution skews toward 0, and as η values become large, the distribution skews toward 1. With the beta-distribution, more general type of form error including non-symmetric distribution can be modeled. We include the form error of the surface for the investigation of the sampling and uncertainty relation.

ANALYTIC APPROXIMATION OF THE UNCERTAINTY FUNCTION

The functional relationship between the uncertainty and the measured point characteristics can be represented by the standard error function in terms of the form error, sample size, and the geometry. Since an extreme fit is a numerical solution, the derivation of the functional relationship is not straightforward. Thus, an approximation is proposed and shown herein for the flatness example. We employ an order statistics to model the flatness evaluation. The order statistics provides statistical properties of ranked elements including the maximum and the minimum. The extreme points can be modeled as the ranked elements in a certain geometric domain. The standard error of an extreme fit feature depends on the variance of the extreme points when sampled from an unknown true surface. The variance of the extreme points is determined by the variance of order statistics elements. Thus, after the variance of order statistic elements is derived, the standard error of the extreme fit can be represented as a function of sample size.

Suppose the statistic x is distributed by a probability density function f(x). Let us denote the cumulative probability distribution as F(x). From the distribution, a random sample of n data are taken, and the sample data are denoted as xi, i = 1, ... n. The random sample is a set of measured data in a certain geometric domain. Let us define xm as the minimum of xi, and xM as the maximum of the xi. Then the range of x_i is the difference between the maximum and the minimum as

$$w = x_M - x_m, \quad 0 < w < \infty \tag{9}$$

If x_i is the normal directional variation for i-th point of measurement data from a plane, the range w is an approximation of the flatness value. The standard error of flatness evaluation can be derived from the variation of the range w. From order statistics (Balakrishanan19), the joint density for xm to be the minimum and for xM to be the maximum is:

$$g(x_m, x_M) = n(n-1)\{F(x_M) - F(x_m)\}^{n-2} f(x_m) f(x_M) \qquad (10)$$

The probability density g can be expressed as a function of the range w and the minimum xm as

$$g(x_m,w) = n(n - 1)\{F(x_m + w) - F(x_m)\}^{n-2} f(x_m)f(x_m + w) \qquad (11)$$

The probability density for the range w can be derived as integrating (11) with respect to xm over all possible range as

$$h(w) = n(n-1)\int_{-\infty}^{\infty} \{F(x_M + w) - F(x_m)\}^{n-2} f(x_m) f(x_M + w) dx_m \qquad (12)$$

(12) represents the probability density of a variable w to be the range of the order statistics. Thus, it is the probability density of the flatness value when n data points are taken from a surface that has the probability density f(x). Then, the mean of w is

$$\mu_w = E[w] = \int_0^{\infty} w \cdot h(w)\, dw \qquad (13)$$

and the variance is

$$Var(w) = E[(w-\mu_w)^2] = \int_0^{\infty} (w-\mu_w)^2 \cdot h(w)\, dw \qquad (14)$$

From (14), the standard error of flatness can be expressed as a function of the number of data points n. Let us consider the following example. Suppose a surface is modeled by the beta distribution with the exponents η and λ unity, then the probability density of the noise x is a simple uniform distribution as

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases} \qquad (15)$$

The functional relationship between the number of measurement points and the standard error of the evaluation can be analytically modeled. By (12), the probability density of the range w can be determined as

$$h(w) = n(n-1)\int_0^{1-w} w^{n-2} dx_m$$

$$= n(n - 1)w^{n - 2}(1 - w),\ 0 < w < 1 \qquad (16)$$

Substituting (16) into (13) gives the mean of the range as

$$\mu_w = n(n-1)\int_0^1 w^{n-1}(1-w)dw = n(n-1)\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \frac{n-1}{n+1} \qquad (17)$$

and the variance is given as

$$\mathrm{Var}(w) = n(n-1)\int_0^1 \left(w - \frac{n-1}{n+1}\right)^2 \cdot w^{n-2} \cdot (1-w)dw$$

$$= \frac{2(n-1)}{(n+2)(n+1)^2} \qquad (18)$$

Thus, the standard error of w can be expressed as

$$se(w) = \sqrt{\mathrm{Var}(w)} = \sqrt{\frac{2(n-1)}{(n+2)(n+1)^2}} \qquad (19)$$

(19) represents the uncertainty of the flatness evaluation with respect to the sample size n when the distribution is uniform. According to the formulation, for 100 points, the standard error of the flatness evaluation is 0.0138. Since the form error distribution is uniform as (15), the standard error is 1.38% of the evaluated flatness value. If the standard error is to be smaller than 1 percent, the number of sample must be more than 140 points. This approximation yields optimistic results because the rigid body transformation of the measured points is not considered.

For the general case, the formulation of the standard error is difficult to derive analytically. The terms in (12), (13), and (14) are in integral form which is difficult to analytically formulate. For the general beta distribution as defined in (4), the integration for (12), (13), and (14) are not available. Thus, the analytic formulation has limited applications. For a more general case, an experimental approach must be investigated.

EXPERIMENTAL APPROACH

For a given measurement condition, the functional relationship between the standard error associated with evaluation and the measurement parameters needs to be derived. The standard error represents the uncertainty of the form error evaluation. Since such a functional relationship cannot be generally formulated as an analytic form, it can be estimated by an experimental approach. A neural network method is employed to model the functional relation. A neural network is trained to learn the pattern from a set of known data. After the network is trained, it is able to output the standard error for a new input data. Then the network represents the functional relationship between the input parameters and the output parameter. The advantage of this approach is that the explicit form of the function is not necessary. If the function is approximated by a multivariate regression, the explicit form of the function, such as a polynomial and the order of the polynomial must known beforehand. A neural network can represent a function without knowing the explicit form and handles more general types of functions.

In order to build a network that represents the functional relationship, a training data set must be provided. The training data set includes various measurement conditions. In each case of the measurement, the possible standard error of the evaluated value must be provided. The standard error is evaluated by iterating the process of sampling from a given known surface and by evaluating the extreme fit. Then, from repetitive evaluations of the extreme fit, the standard error of the evaluation is determined. The set of the measurement parameters including the geometry, form error, the number of samples, and the evaluated standard error are used for the input and the output of the network. From different training sets, the network picks up the functional relationship between the parameters. A flatness model is demonstrated as follows.

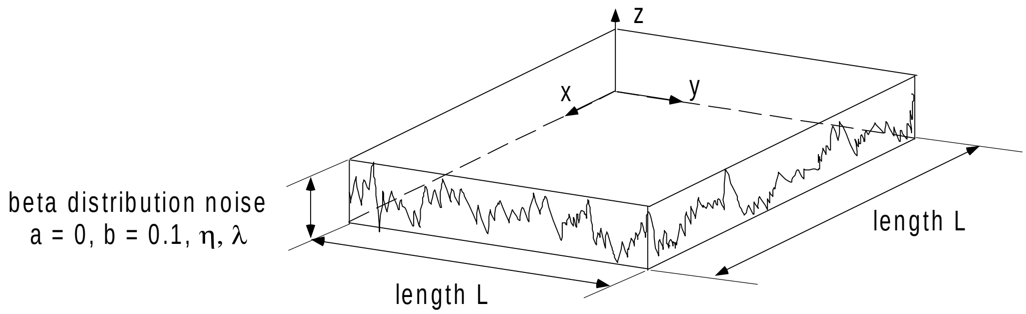

Figure 7 Test Model.

The model to be tested is an L by L flat square represented by x-y plane. In order to simulate the form error of a real surface, a beta distribution noise is added to the surface. The beta distribution is able to model a general type of stochastic error that occurs in engineering surfaces. The range of the form error is set as a = 0., b = 0.1 of the beta distribution. Thus, the ideal flatness value of the surface is 0.1. In order to make training data sets, flatness of the surface and the standard error of the flatness must be evaluated. A set of n points are randomly selected from the surface, then flatness is evaluated from the point set. Flatness of the model surface is evaluated by the minimum zone evaluation. When a different set of n points are taken, the evaluated flatness value changes slightly. The standard error of the flatness is determined as the samples are repetitively resampled, Then, the sampling uncertainty is represented by the standard error of the flatness evaluation value.

The parameters that define the model are the geometry parameter, length L, and the noise parameters, h and l of beta distribution. A feedforward network with one hidden layer has been built as shown in Figure 8. The network consists of four inputs and an output. Inputs are model parameters and the sample size. The output is the standard error of flatness value.

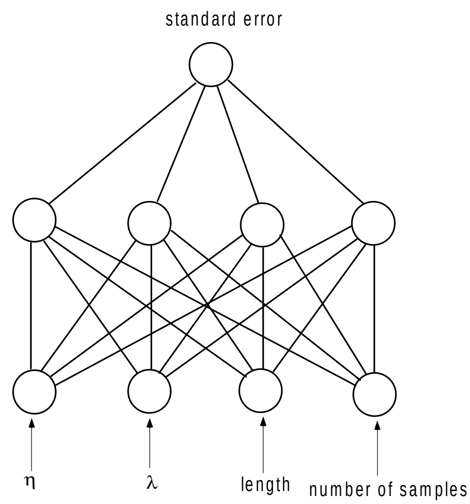

Figure 8 Network Model.

The training data have been generated as shown in Table 1. The ranges of the parameters for the training data set are shown as the minimums and the maximums of the parameter values. Different parameter values, which increment from MIN to MAX, are tested. The beta distribution parameter spans from 1. to 5.. With different l and h values, the different cases of the surface form error are modeled. When l and h are both 1., the beta distribution is equivalent to uniform distribution. In each case of the beta distribution, different number of measured data samples are taken. The number of samples are from 20 to 200, which is a typical CMM measurement size. Then, the flatness value is evaluated. In order to evaluate the standard error of the evaluation, each case is iterated 100 times and from the iteration, the standard error of the flatness value is derived.

	MIN value	Max value	number of increments
η	1.	5.	5
λ	1.	5.	5
l	10.	1000.	3
number of sample	20	200	4

Table 1 Training Data Set for Neural Network.

RESULTS

The neural network was implemented using Stuttgart Neural Network Simulator (Zell, etc. al20), version 4.1. The network was trained by the backpropagation method using the momentum algorithm. The learning rate α = 0.3 and the momentum term m = 0.4 were used. The network was trained in 50,000 cycles with the learning data. The square sum of error defined in (17) is 0.3535, and the mean square error is 0.0026. Thus, the root mean square error is 0.0508. The approximated function has about 5% error in the output. The error is acceptable considering the fact that the standard error contains a certain level of random noise because it has been evaluated from a finite number of samples. Note that if the network were trained to fit all the outputs exactly, it may be over constrained so that it generates unnecessary fluctuations.

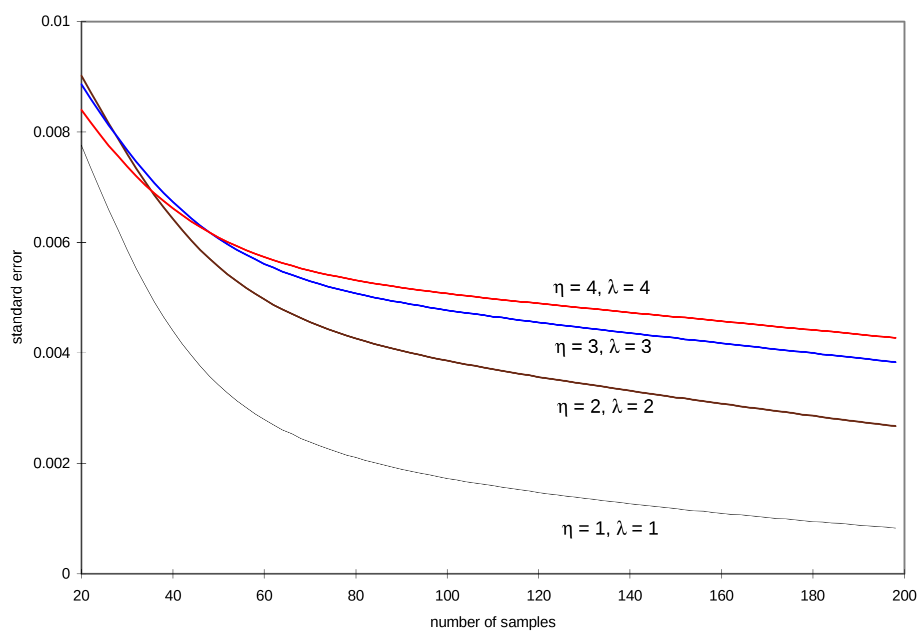

Figure 9. Standard Error With Respect to Different Sizes of Form Error vs. Number of Samples.

Since the function is in the form of a neural network, the explicit mathematical form is not known, and the result must be presented in a numerical form. Figure 9 is a plot of the function with respect to the sample size. In Figure 9, the standard error monotonically decreases with respect to the sample size. For the distribution with l = 1, h

= 1, which is a uniform distribution with range 0.1, the standard error is below 1x10-3 with more than 170 points. Considering the magnitude of flatness (for l = 1, h = 1), 0.1, the standard error is less than 1%. Compared to the analytic approximation, the experimental results yield higher standard error for the same number of points. It is mainly due to the simplification in the analytic approximation that ignored the rigid body transformation of the surface in the evaluation.

An interesting observation is that the standard error increases as the pair h,l increase in beta distribution. As h and l increase, the probability distribution is more dense at the center as the bell shape becomes steep as shown in Figure 6. Thus, as h and λ increase, even though the variance of the form error decreases, the standard error of flatness increases. This is consistent with the observation that the standard error of flatness evaluation from a surface with normally distributed form error is larger than the standard error from a surface with uniformly distributed form error.

This can be explained by the distribution of the maximum and minimum points. Even though the maximum and minimum points for initial noise distribution may not be the extreme points determined by the minimum zone evaluation, the distribution of the maximum and the minimum points is closely related to the magnitude of the minimum zone. As the noise is distributed more toward uniform distribution, the maximum and the minimum points from n samples are fairly close to the limits. Thus, the range between the maximum and the minimum is more consistent. If the noise distribution is more dense at the center, the average range between the maximum and the minimum is decreased but the variation of the range is increased. Thus, as h,l increases, the standard error increases as illustrated in Figure 9. As a result, the normal distribution yields larger standard error than the uniform distribution.

This tendency is reversed as the number of samples decreases as shown in Figure 9. If the number of samples is small, the samples are not fully distributed in the range. The discrepancy between extreme points of the sample and the true surface is larger. As the distribution becomes more uniform the discrepancy increases as the number of points becomes small. For steep bell shaped distributions, the points are mostly distributed at the center, which has less sensitivity to the number of points. Thus, compared to uniform distribution, the variation of the range is small.

Figure 10 shows the trend of the standard error with respect to the skewness of the distribution. As the coefficient of skewness increases, the standard error increases. Since the flatness value is theoretically symmetric with respect to the skewness direction, either in the positive direction or the negative direction, the sign of skewness should not affect the standard error of the flatness value. However, for actual measurement data, the sign of the skewness is important. A positive skewness value for a form error represents valleys in the surface as shown in Figure 5. For a contacting surface such as a bearing surface, the positive skewness may be acceptable because the valleys of a surface may have a minimal affect on the functionality of the part. Also the high frequency noise cannot be measured by a relatively large probe. On the contrary, a negative skewness means spikes on the surface, which are critical deformations. The spikes are more apparently measured, and the standard error of the evaluation increases.

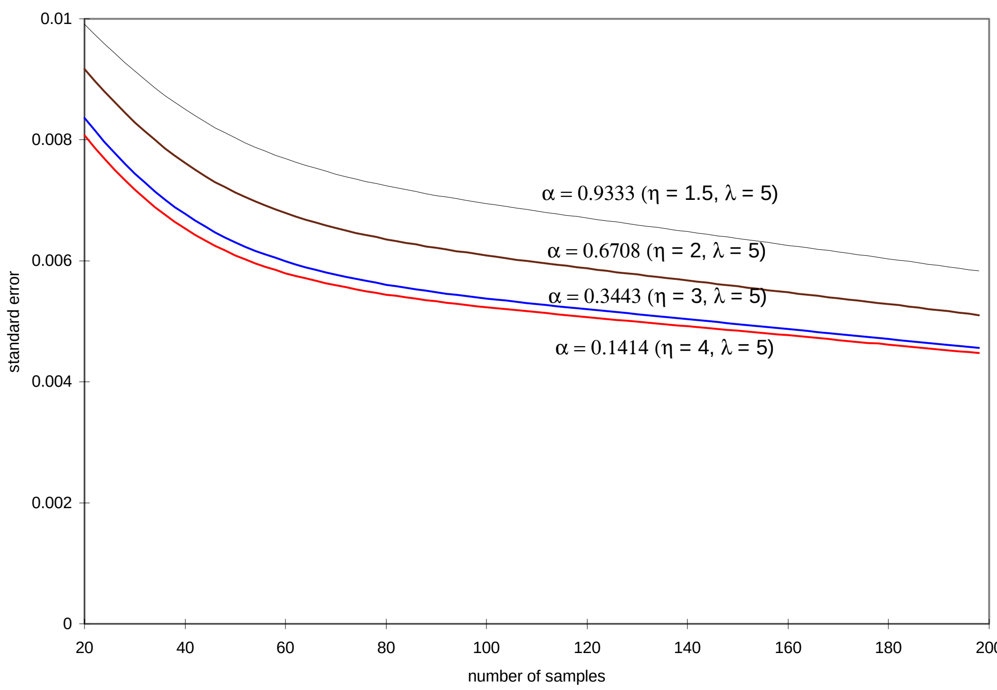

Figure 10 Standard Error With Respect to Skewness of Distribution vs. Number of Samples

An interesting observation is the relation between the geometry of the model and the standard error. As shown in Figure 11, there is very weak correlation between the standard error and the geometry size. Throughout the range of the length used, from 20 to 1000, Figure 11 shows almost consistent standard error. This result indicates the density of the measured points has little affect on the uncertainty. For a 200 number point case, the areal density varied from 0.5 points per unit area to 2x10-4 points per unit area, yet they yield almost similar standard error. Thus, it is the number of measurement points, not the density of the points, that is influential to the uncertainty.

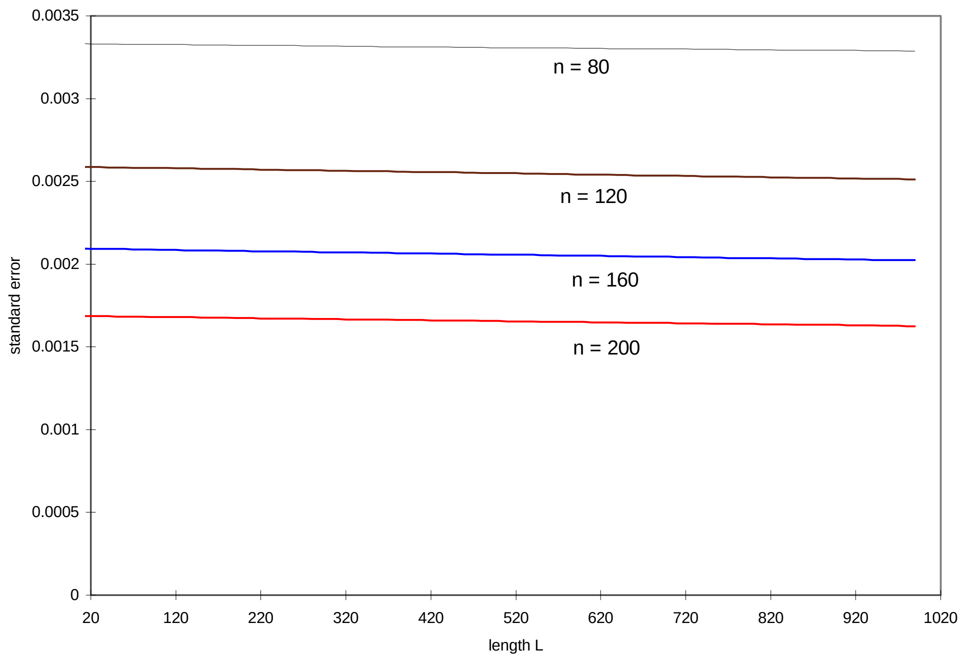

Figure 11 Standard Error and the Size of Measurement Region ($\eta = \lambda = 1.5$).

CONCLUSIONS

The results show how the property of a measured point set affects the uncertainty of an extreme fit evaluation. The established numerical model represents how the standard error of the evaluation changes with respect to the number of measured points. The results indicate that the distribution of the form error is critical for the standard error as well as the number of points. When the distribution approaches more toward a bell shape, which resembles a normal distribution, the standard error of the extreme fit evaluation tends to increase. When the skewness of the form error increases, the standard error also tends to increase. However, the size of measurement area has little effect on the standard error. Thus, the density of the measurement data is not critical for the uncertainty.

The factor that has not been considered in this paper is the distribution of measurement points on the measurement plane. The distribution of measurement points is regarded as uniform in this paper, because the flatness evaluation is a form error evaluation where the probability of such a variation being observed is relatively uniform compared to a systematic deformation characterization. Non-uniform distribution is more effective for a non-uniform variation, which is a systematic deformation. If the evaluation is to characterize a systematic deformation, a non-uniform distribution of measurement points needs to be considered.

For practical measurement situations, the number of measured points needs to be determined before the measurement. The results of this work unfortunately indicate that the uncertainty results depend significantly on the distribution of the form error. This does not lend itself well to determine the number of measurement points needed to achieve a specified level of uncertainty. Since the distribution of form error is unknown before the measurement, it is difficult to predict the uncertainty of the evaluation based on the number of points. Thus, the knowledge of the form distribution must be provided. Such a fact unfortunately indicates that dimensional inspection planning must be processed case by case.

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